FAR BEYOND

MAT122

Logarithmic Derivatives
Part II

Stony Brook University

Log Laws - recall

ex. fully expand $\ln\left(\frac{x+1}{\sqrt{1-3x}}\right)$

use log's Quotient Rule

$$= \ln(x+1) - \ln\sqrt{1-3x}$$

$$= \ln(x+1) - \ln(1-3x)^{1/2}$$

$$= \ln(x+1) - \frac{1}{2}\ln(1-3x)$$

Log Laws

$$\ln(MN) = \ln M + \ln N$$

$$\ln\left(\frac{M}{N}\right) = \ln M - \ln N$$

$$\ln x^P = P\ln x$$

Derivative of $y = ln\ x$ with Log Laws

$$\frac{d}{dx}\ln x = \frac{1}{x}$$

$$\frac{d}{dx}\ln\left(\frac{x+1}{\sqrt{1-3x}}\right) = \frac{d}{dx}\left[\ln(x+1) - \frac{1}{2}\ln(1-3x)\right]$$

use log laws to expand

$$= \frac{d}{dx}\ln(x+1) - \frac{1}{2}\frac{d}{dx}\ln(1-3x)$$ DO: take both derivatives

$$= \frac{1}{x+1}\cdot 1 - \frac{1}{2}\frac{1}{1-3x}\cdot(-3)$$

$$= \frac{1}{x+1} + \frac{3}{2(1-3x)}$$ get LCD

$$= \frac{1}{x+1}\cdot\frac{2(1-3x)}{2(1-3x)} + \frac{3}{2(1-3x)}\cdot\frac{x+1}{x+1}$$ DO: combine to single fraction

$$= \boxed{\frac{-3x+5}{2(x+1)(1-3x)}}$$

Log Laws

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$$\ln x^P = P\ln x$$

Derivative with Log Laws - Do

Do: differentiate $y = \ln[(5x^2+1)^4(x^4-3)^6]$

Log Laws

$$\ln(MN) = \ln M + \ln N$$

$$\ln\left(\frac{M}{N}\right) = \ln M - \ln N$$

$$\ln x^P = P \ln x$$

$$\frac{d}{dx}\ln x = \frac{1}{x}$$

$$= \boxed{\frac{40x}{5x^2+1} + \frac{24x^3}{x^4-3}}$$

More Practice w Log Laws

ex: differentiate $y = \ln\left(\dfrac{x^{3/4}\sqrt{x^2+1}}{(3x+2)^5}\right)$ use Quotient Rule to expand

$$= \ln\left(x^{3/4}\sqrt{x^2+1}\right) - \ln(3x+2)^5$$

use Product Rule to expand

$$= \ln\left(x^{3/4}\right) + \ln\left(\sqrt{x^2+1}\right) - \ln(3x+2)^5$$ now use Power Rule for exponents

$$= \frac{3}{4}\ln x + \frac{1}{2}\ln\left(x^2+1\right) - 5\ln(3x+2)$$

$$\therefore y' = \frac{3}{4}\cdot\frac{1}{x} + \frac{1}{2}\cdot\frac{1}{x^2+1}\cdot 2x - 5\cdot\frac{1}{3x+2}\cdot 3$$

$$= \boxed{\frac{3}{4x} + \frac{x}{x^2+1} - \frac{15}{3x+2}}$$

Log Laws

$$\ln(MN) = \ln M + \ln N$$

$$\ln\left(\frac{M}{N}\right) = \ln M - \ln N$$

$$\ln x^P = P\ln x$$

$$\frac{d}{dx}\ln x = \frac{1}{x}$$